

Riemann boundary-value problem for holomorphic matrices on non-rectifiable curve

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Abstract

© 2017, Allerton Press, Inc. We study the Riemann boundary-value problem on non-rectifiable curves for holomorphic matrices with Fokas-Its-Kitaev asymptotics by means of the Cauchy transforms of certain distributions with supports on that curves. The main results concern the existence of solutions of sufficiently large degree.

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Keywords

holomorphic matrix, non-rectifiable curve, Riemann boundary-value problem

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